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Soft-gluon resummation for *H*-production: Methods and results

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I will discuss soft-gluon resummation for the Higgs total cross section. Different methods

- based on the same factorization formula,
- with same N^3LL+NNLO accuracy,

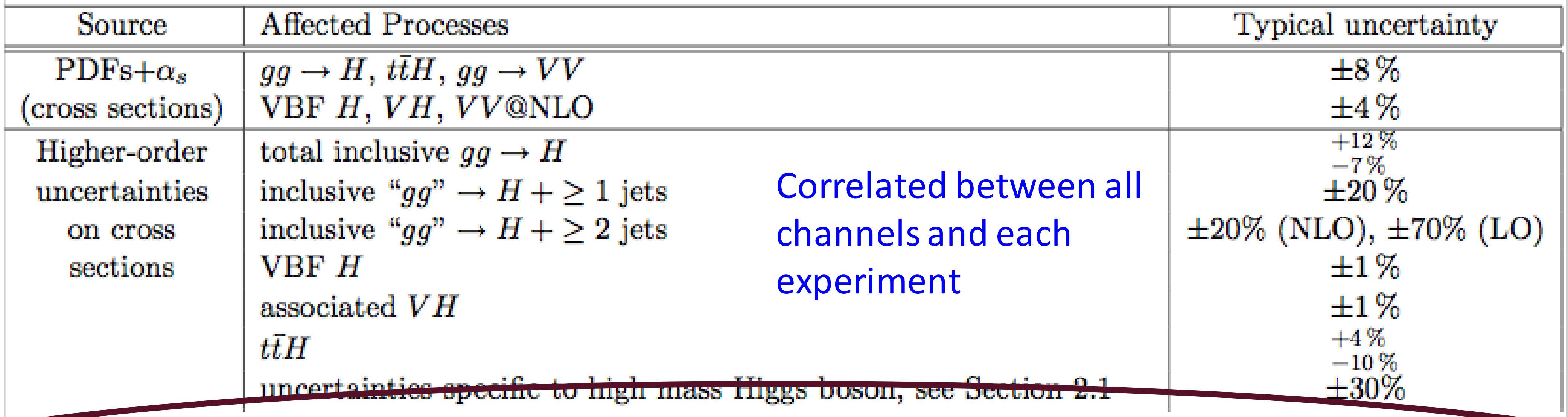
can give fairly (~ 5-10%) different cross sections.

Different choices of

- expansion parameters,
- scale setting prescriptions

Each Higgs production also comes with different relative uncertainties

At LHC



Source	Affected Processes	Typical uncertainty
PDFs+α_s (cross sections)	$gg \to H$, $t\bar{t}H$, $gg \to VV$	$\pm 8\,\%$
	VBF H, VH, VV@NLO	$\pm 4\,\%$
Higher-order uncertainties on cross sections	total inclusive $gg \to H$	$^{+12\,\%}_{-7\,\%}$
	inclusive "gg" $\to H + \geq 1$ jets	$\pm 20\,\%$
	inclusive "gg" $\to H + \geq 2$ jets	$\pm 20\%$ (NLO), $\pm 70\%$ (LO)
	VBF H	$\pm 1\,\%$
	associated VH	$\pm 1\,\%$
	$t\bar{t}H$	$^{+4\,\%}_{-10\,\%}$
	uncertainties specific to high mass Higgs boson, see Section 2.1	$\pm 30\%$

Correlated between all channels and each experiment

gg → H uncertainties are largest despite tremendous set of calculations :

QCD radiative corrections at NLO
QCD corrections NNLO
QCD soft-gluon resummation NNLL
EWK corrections NLO
top and bottom loop corrections up NLO
above 400 GeV, line shape unknown

Details & references in CMS+ATLAS combination note

State of the art predictions

Fixed order:

Anastasiou, Bühler, Herzog, Lazopoulos, 1107.0683

http://www.phys.ethz.ch/~pheno/ihixs/

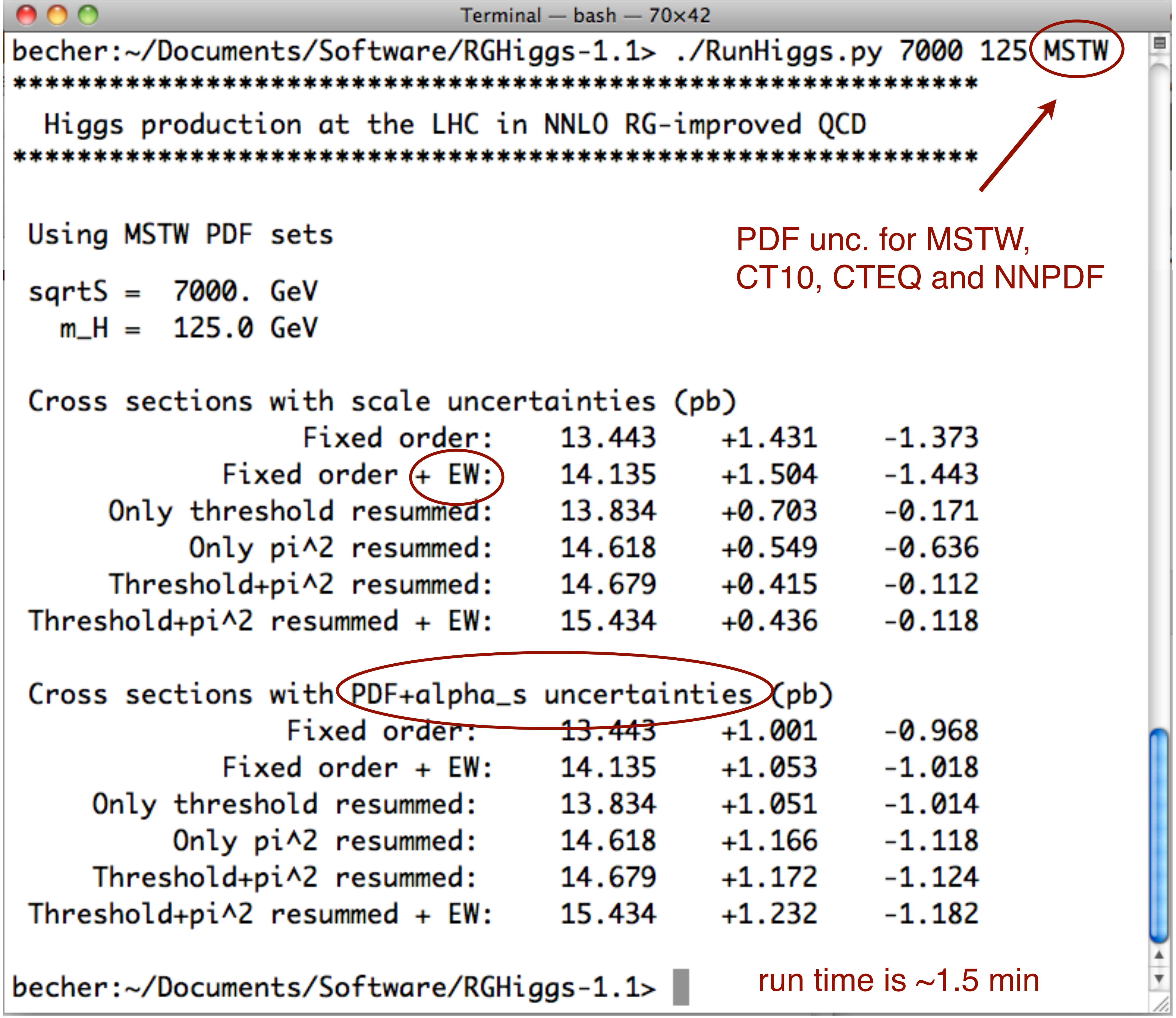
Resummed:

de Florian, Grazzini, 0901.2427

http://theory.fi.infn.it/grazzini/hcalculators.html

Ahrens, TB, Neubert, Yang, 1008.3162

http://projects.hepforge.org/rghiggs/



```
becher:~/Documents/Software/RGHiggs-1.1> ./RunHiggs.py 7000 125 MSTW
************************************************************

  Higgs production at the LHC in NNLO RG-improved QCD
************************************************************


 Using MSTW PDF sets

 sqrtS =  7000. GeV
   m_H =  125.0 GeV

 Cross sections with scale uncertainties (pb)
                  Fixed order:    13.443    +1.431    -1.373
             Fixed order + EW:    14.135    +1.504    -1.443
      Only threshold resummed:    13.834    +0.703    -0.171
           Only pi^2 resummed:    14.618    +0.549    -0.636
      Threshold+pi^2 resummed:    14.679    +0.415    -0.112
 Threshold+pi^2 resummed + EW:    15.434    +0.436    -0.118

 Cross sections with PDF+alpha_s uncertainties (pb)
                 Fixed order:     13.443    +1.001    -0.968
             Fixed order + EW:    14.135    +1.053    -1.018
     Only threshold resummed:     13.834    +1.051    -1.014
          Only pi^2 resummed:     14.618    +1.166    -1.118
     Threshold+pi^2 resummed:     14.679    +1.172    -1.124
 Threshold+pi^2 resummed + EW:    15.434    +1.232    -1.182

becher:~/Documents/Software/RGHiggs-1.1> 
```

PDF unc. for MSTW,
CT10, CTEQ and NNPDF

run time is ~1.5 min

Results for the cross section

	σ [pb]	scale unc. $\Delta\sigma$ [%]
iHixs	15.37	+9/-8
deFG	15.40	+7/-8
RGHiggs	15.43	+3/-1

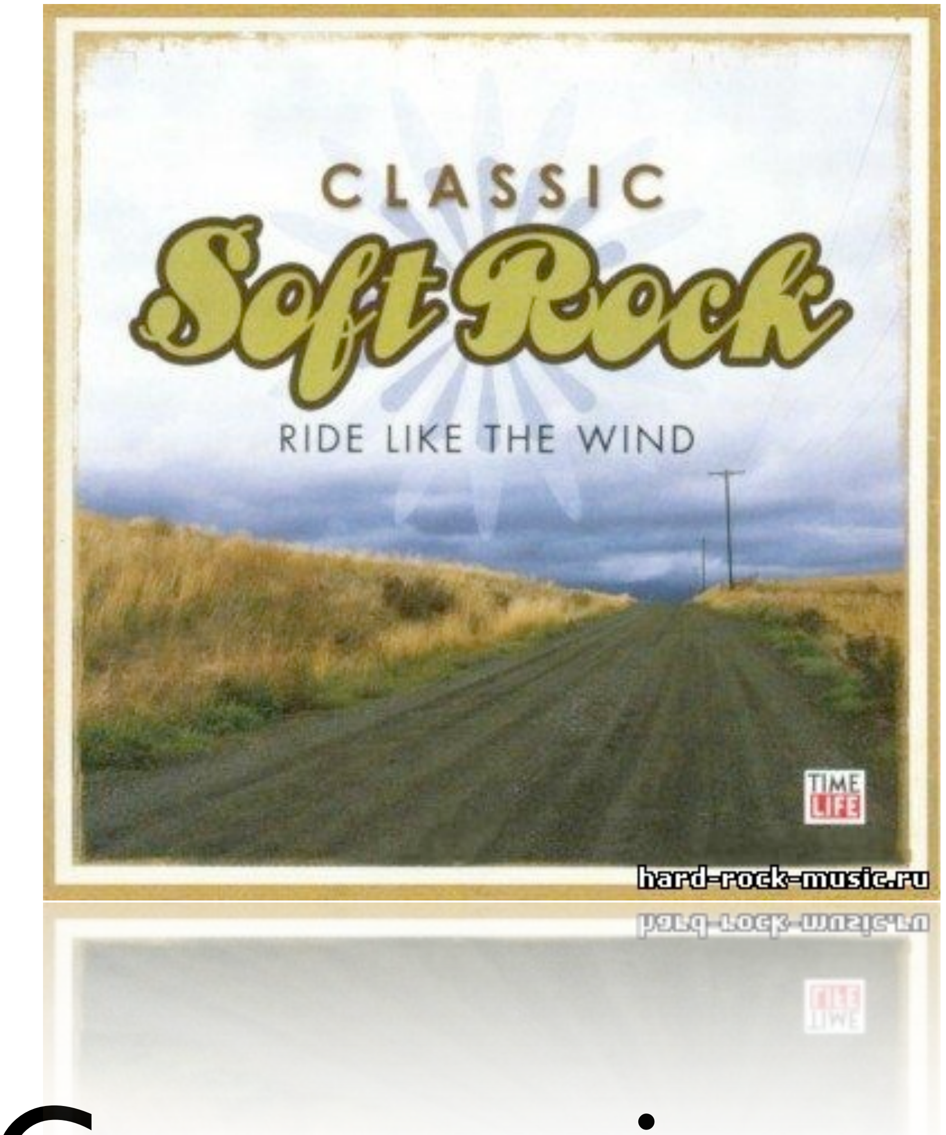
for m_H=125 GeV, LHC 7TeV, m_t=173.1 GeV, m_b=4.2 GeV

- Based on MSTW08NNLO.
  - ± 8 % PDF + α_s uncertainty @ 90% CL
  - ± 4 % PDF + α_s uncertainty @ 68% CL
  - PDF4LHC prescription gives +8/-7% unc.
- *Numerically*, there is excellent agreement for σ .

Differences

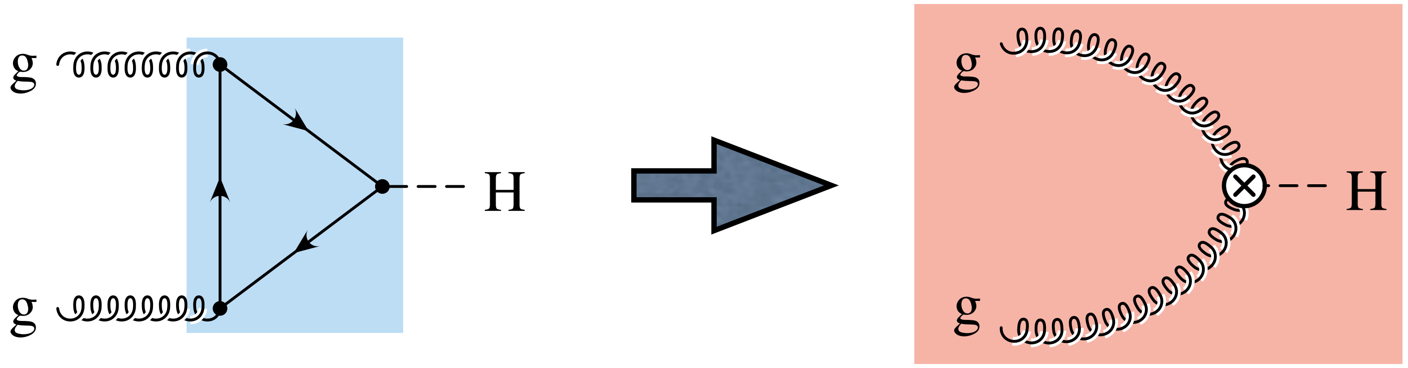
Several differences in resummed results hide behind the numerical agreement:

- We find that soft-gluon resummation alone increases cross section by 3%, dFG find 8%. More than a factor of 2 difference in the resummation itself!
- Use different treatment of hard function gives ("π^2 resummation"), yields 9% increase.
  - once this is done, soft-gluon resummation itself becomes negligible
- iHixs uses $\mu = M_H/2$: σ is 10% larger than at $\mu = M_H$.



Common issues in soft-gluon resummation

Common first step: integrate out the top



$$\mathcal{L}_{\text{eff}} = C_t(m_t^2, \mu^2)\, \frac{H}{v}\, \frac{\alpha_s(\mu^2)}{12\pi}\, G_{\mu\nu,a}\, G_a^{\mu\nu}$$

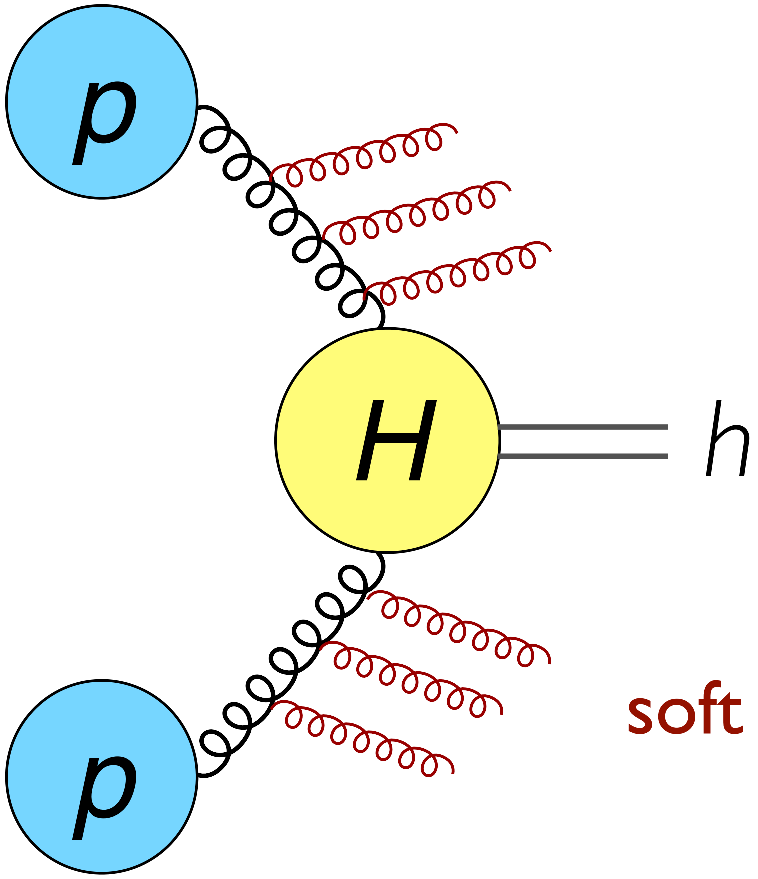
For $m_H \ll 2m_t$ we can integrate out the top quark, i.e. replace the SM by an effective theory with $n_f = 5$.

Calculations in EFT are much simpler. One loop and one scale less. NNLO results only available in EFT.

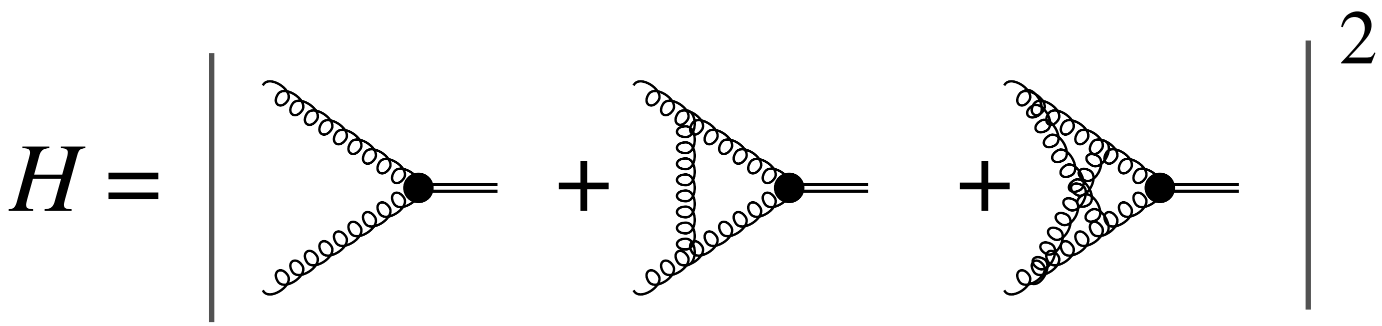
C_t known to NNNLO, excellent convergence. Power corrections $(m_H/m_t)^2$ are small for light Higgs.

Common: factorization theorem

Sterman '87, Catani & Trentadue '88



soft radiation



$$\tau = \frac{m_H^2}{s} \approx 0.0003$$

$$H(m_H^2, \mu) \int_\tau^1 S(\sqrt{\hat{s}}(1-z), \mu) ff_{gg}(\tau/z, \mu)$$

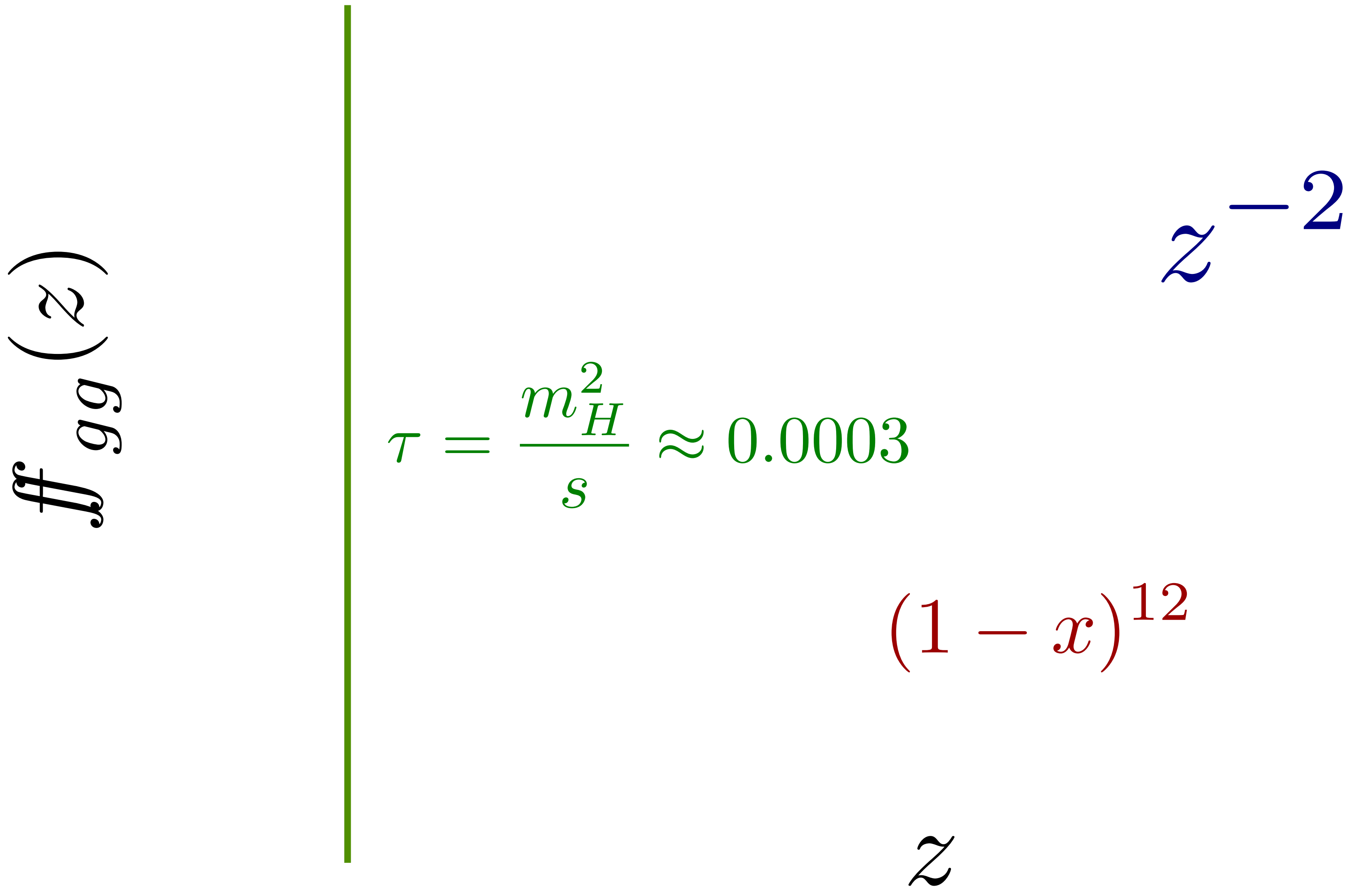
hard function

soft function

parton luminosity

Scale of soft radiation is lower than m_H: large (?) pert. logarithms.

Parton Luminosity



Fall-off is not very strong. Will find typical scale of radiation is of order $M_H/2$.

Threshold dominance?

parton luminosity:

a $\quad$ $ff_{gg}(z) \sim z^a$

z

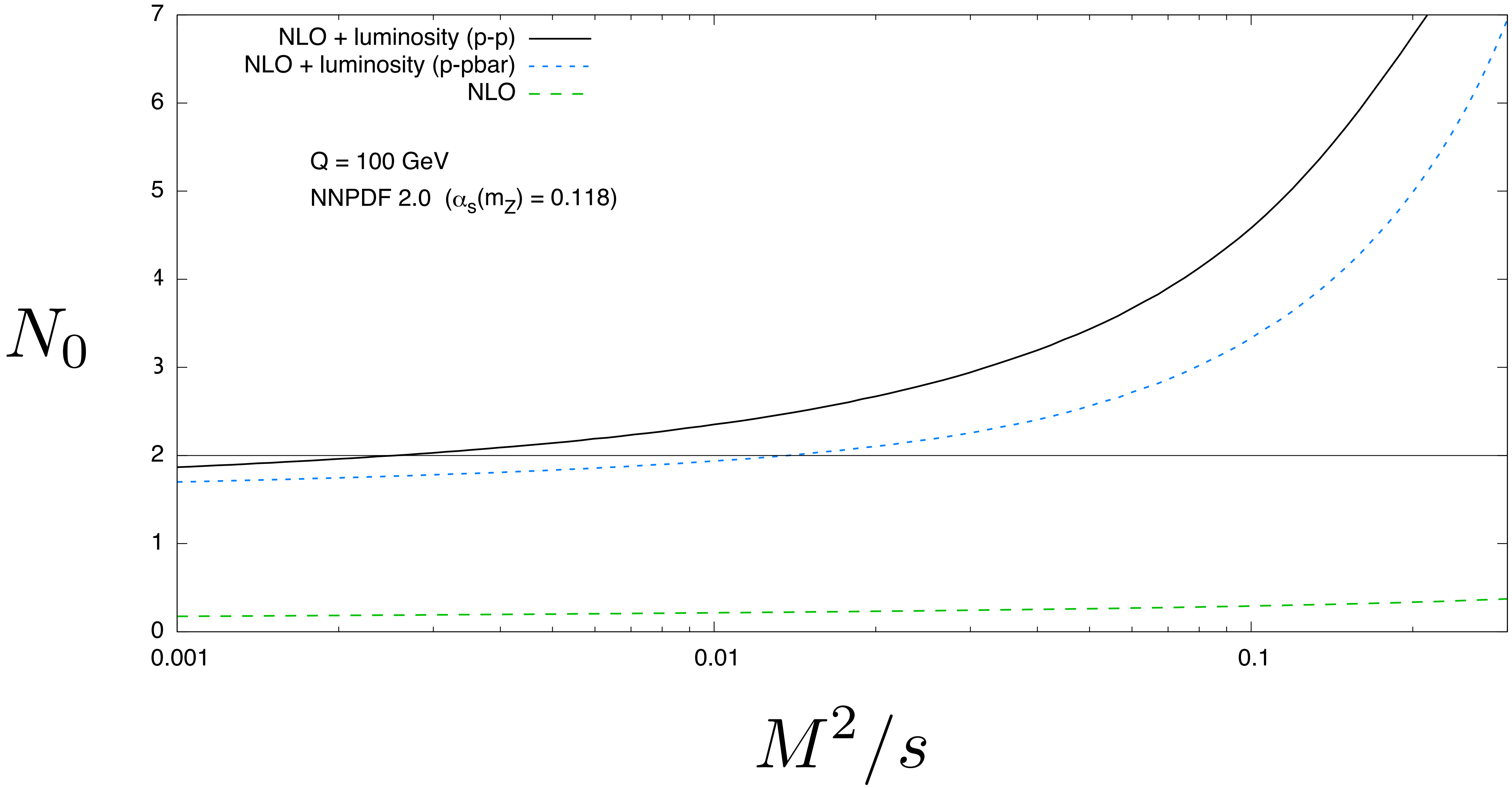
$$\sigma \approx \sigma_{\rm Born} \int_0^1 dz\, z^{a-1}\, C(z, m_t, m_H, \mu_f)\,; \qquad \sigma_{\rm Born} = \sigma_0\, ff_{gg}(\tau, \mu_f)\,,$$

At $z = \tau = M_H^2/s \sim 0.0003$ the fall off is a=2.5.

→ Threshold region is *not* strongly enhanced.

Moment space

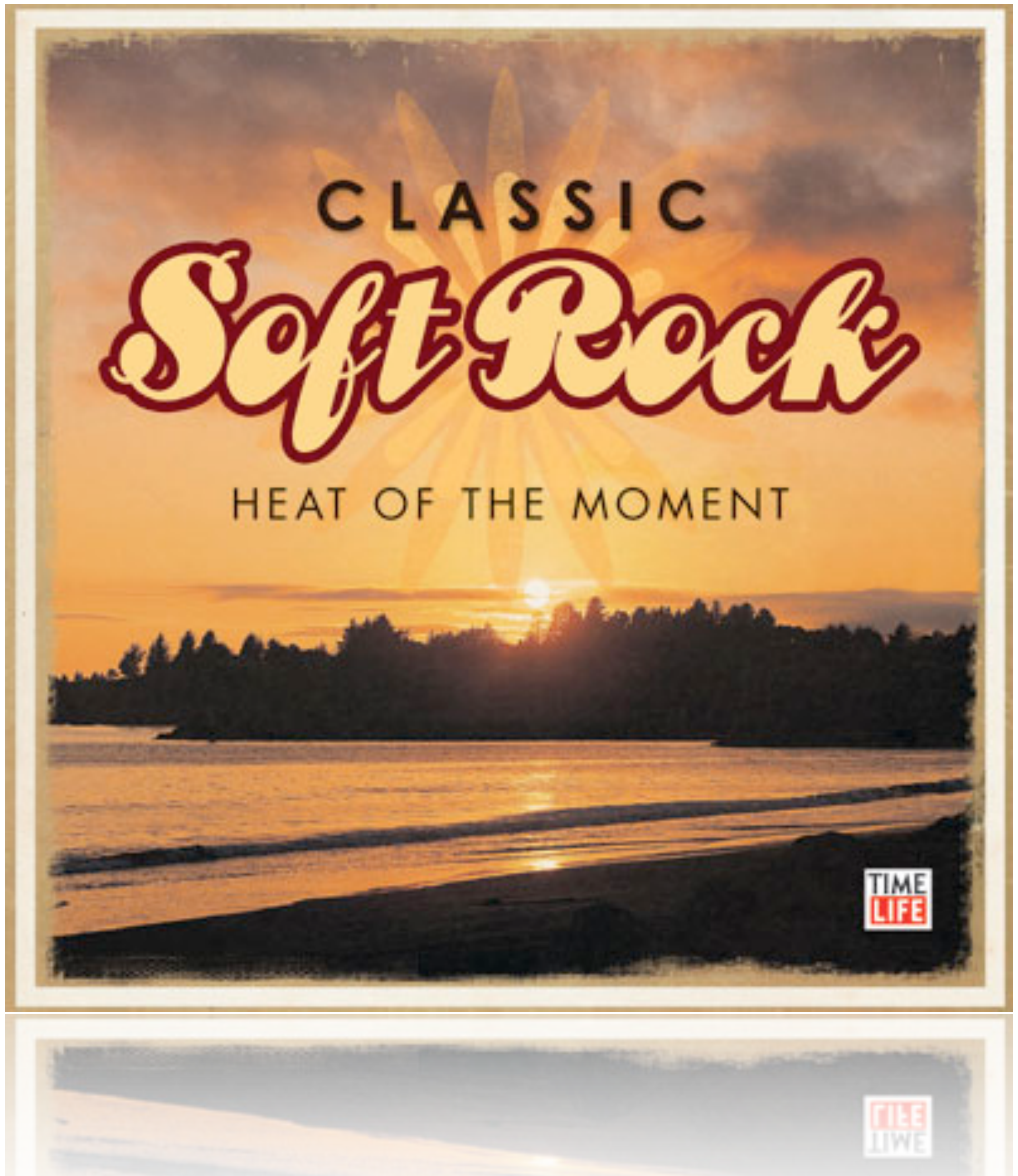
Bonvini, Forte Ridolphi 1009.5691



Same picture: typical moment is second moment of cross section. (Note: Plot is for Drell-Yan not Higgs.)

Upshot

- For Higgs at LHC, harder emissions are not strongly suppressed by PDFs.
- Expansion around soft limit has an expansion parameter ~1/2
  - Exact choice of expansion parameter (or space in which expansion is performed) matters.
  - Relatively strong scheme depenence.



Three differences

Differences

1. Integral transform / choice of sing. distribution
    a. Mellin moments
    b. Laplace transform (in E_s)
2. Scale setting for soft emissions
    a. on the partonic level
    b. on the hadronic level
3. Evaluation of for the hard function
    a. time-like
    b. space-like

de Florian and Grazzini   Ahrens, TB, Neubert, Yang

Soft emissions

$$\sigma(\tau) = \sigma_0(\mu_f) \int_\tau^1 \frac{dz}{z}\, C_{gg}(z, m_t, m_H, \mu_f)\, \mathit{ff}_{gg}(\tau/z, \mu_f)$$

Soft emissions give rise to singular distributions in partonic cross section *C*C_{gg}.

$$\frac{2E_s(z)}{M_H}$$

$$\left[\frac{\ln^n(1-z)}{1-z}\right]_+ \quad \text{or} \quad \left[\frac{1}{1-z}\ln^n\left(\boxed{\frac{1-z}{\sqrt{z}}}\right)\right]_+$$

Resummation will predicts singular distribution terms to all orders.

Integral transform

To perform the resummation one takes the Laplace or Mellin moment transform of the cross section.

$$\mathcal{L}_N\big[f(\xi)\big] \equiv \int_0^\infty d\xi\, e^{-\xi N}\, f(\xi)\,,$$

$$\mathcal{M}_N\big[f(\xi)\big] \equiv \int_0^1 d\xi\, (1-\xi)^{N-1}\, f(\xi)\,,$$

- In SCET, we solve RG in Laplace space and then invert analytically.
- Traditional resummation performed in moment space. Numerical inversion at the end.

Singular terms from Mellin inversion

Threshold $z = 1$ corresponds to expansion around $N \to \infty$. Mellin and Laplace space results are the same after expansion.

Difference arises in the inverse transform:

$$\begin{tikzcd}
 & (-\ln N)^{n+1} + \dots \arrow[r] & \left[\frac{\ln^n(1-z)}{1-z}\right]_+ + \dots \\
\left[\frac{\ln^n(1-z)}{1-z}\right]_+ \arrow[ur, "\text{Laplace}"] \arrow[dr, "\text{Mellin}"'] & & \\
 & (-\ln N)^{n+1} + \dots \arrow[r] & \left[\frac{\ln^n(-\ln z)}{-\ln z}\right]_+ + \dots
\end{tikzcd}$$

Relation

$$-\ln z = \frac{1-z}{\sqrt{z}} + \mathcal{O}\left[(1-z)^3\right]$$

The main difference between the two approaches is a factor of $\sqrt{z}$:

$$\frac{\ln^n(-\ln z)}{-\ln z} \approx \sqrt{z} \times \frac{\ln^n \frac{1-z}{\sqrt{z}}}{1-z}$$

power correction at
threshold z=1

such terms do arise in
fixed-order computation

Singular terms to N^3LO

numerical results from L.L. Yang

	LO	NLO	NNLO	NNNLO
full	5.08	6.50	4.05	?
Laplace	5.08	4.86	1.82	-0.65
Laplace E_s	5.08	5.56	3.13	0.430
Mellin	5.08	6.25	4.05	0.942

Factor of 2 difference!

- Large differences among schemes.
- The singular pieces in the Mellin approach are close to the full result.

2. Difference: choice of soft scale

$$\int_\tau^1 S(\sqrt{\hat{s}}(1-z),\mu)\, ff_{gg}(\tau/z,\mu)$$

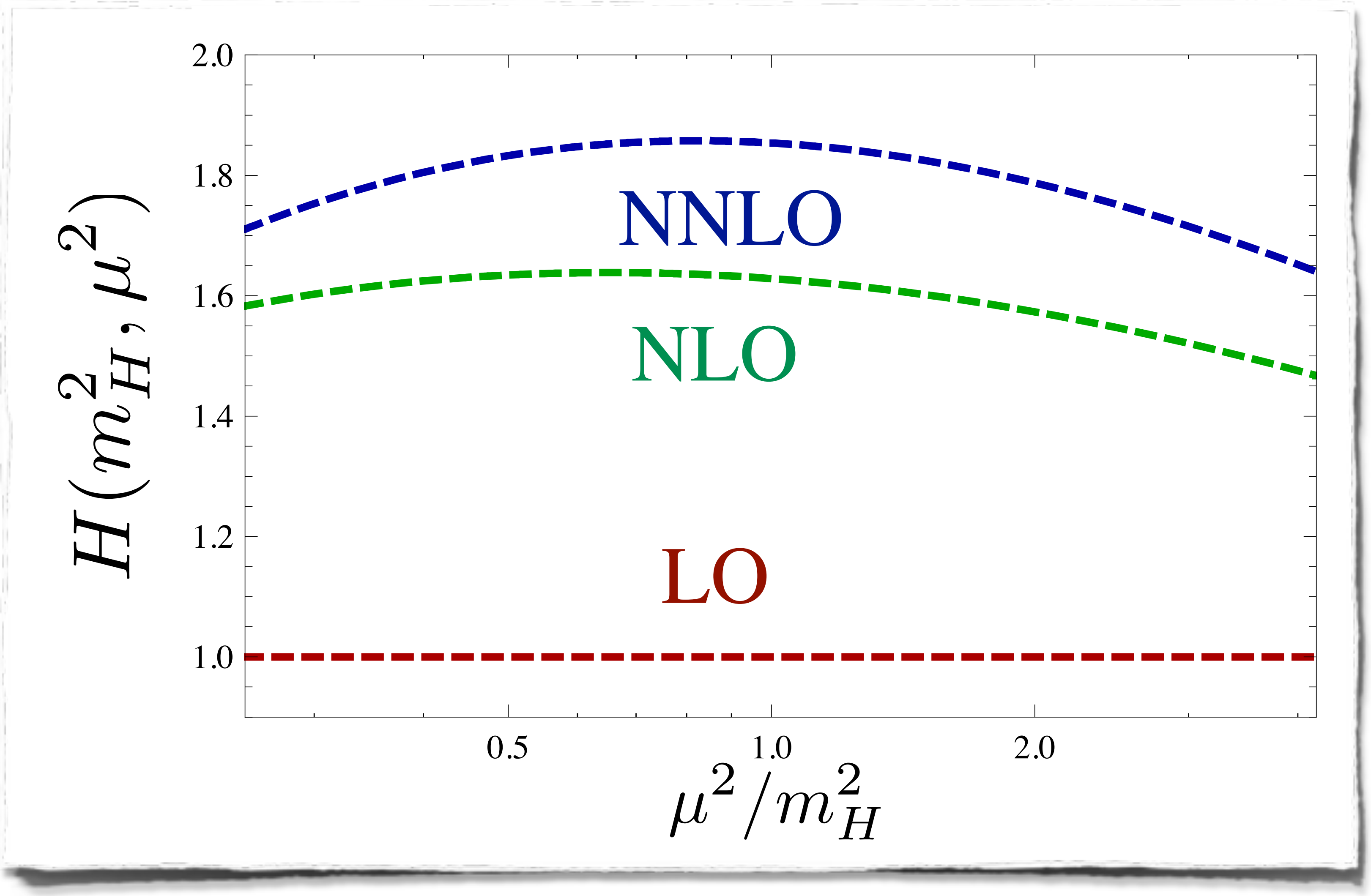
Appropriate scale μ is the soft radiation? Can set scale either at

(a) partonic level: set $\mu = \sqrt{\hat{s}}(1-z)$. Need prescription for Landau pole.

(b) hadronic level: set μ to average energy of soft radiation, determined numerically. Result $\mu \sim M_H/2$.

Numerically, the two prescriptions give similar result [(b) yields 20-50% larger N^{3}LO.]

3. Choice of the hard scale



- Hard function is scale dependent.
- Large corrections for any μ^2 !?!

Scalar form factor

- Hard function $H(m_H^2, \mu^2) = \left| C_S(-m_H^2 - i\epsilon, \mu^2) \right|^2$
- Scalar form factor

$$C_S(Q^2, \mu^2) = 1 + \sum_{n=1}^{\infty} c_n(L) \left(\frac{\alpha_s(\mu^2)}{4\pi} \right)^n, \quad L = \ln(Q^2/\mu^2)$$

$$c_1(L) = C_A \left(\underset{\nwarrow \text{ Sudakov double logarithm}}{-L^2} + \frac{\pi^2}{6} \right)$$

- Perturbative expansions

space-like	$C_S(Q^2, Q^2) = 1 + 0.393\, \alpha_s(Q^2) - 0.152\, \alpha_s^2(Q^2) + \dots$
time-like	$C_S(-q^2, q^2) = 1 + 2.75\, \alpha_s(q^2) + (4.84 + 2.07i)\, \alpha_s^2(q^2)$

Solution

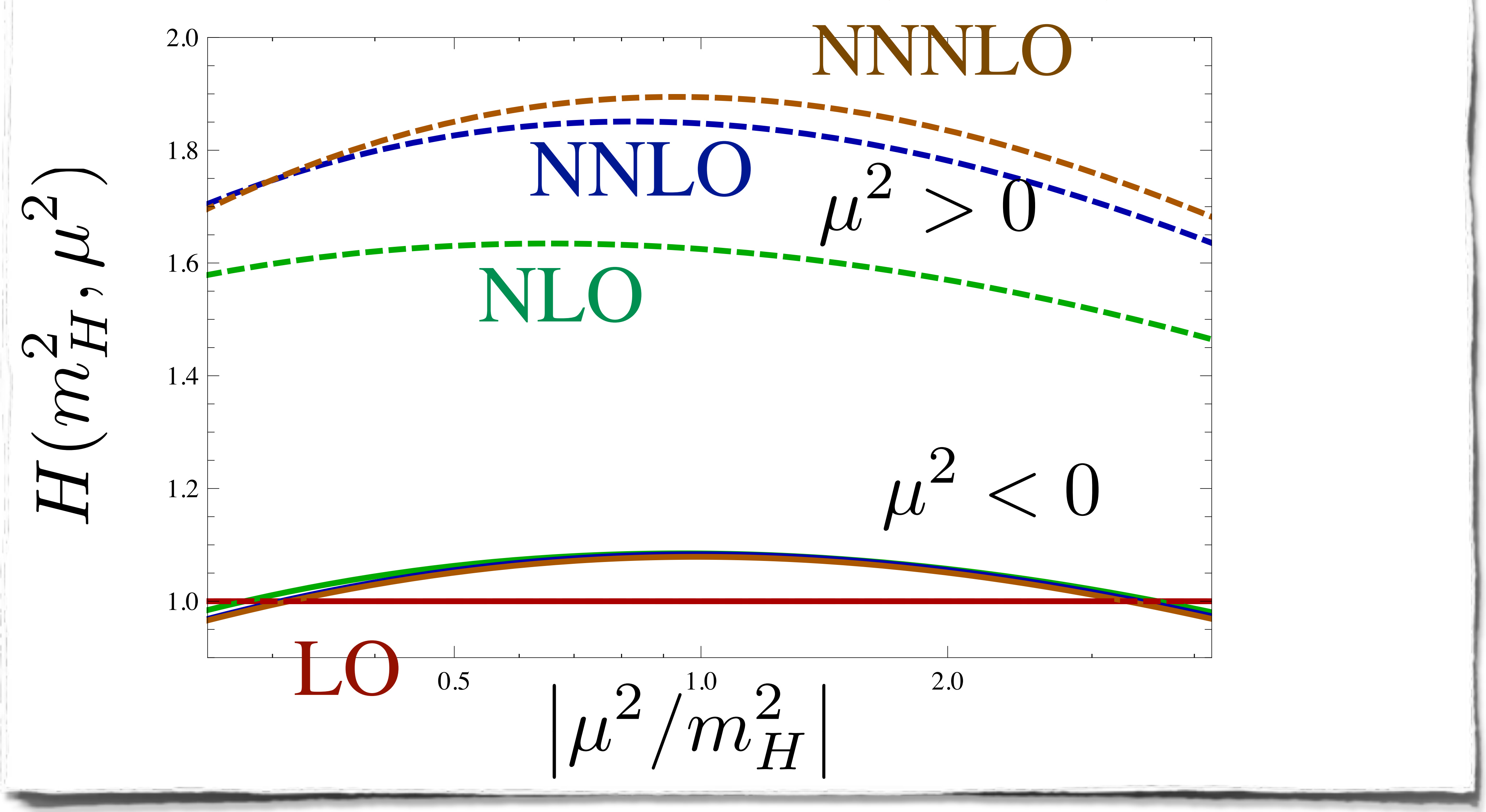
- Reason: $L \to \ln q^2/\mu^2 - i\pi$ and double log's give rise to π^2 terms. Parisi '80
  - Being related to Sudakov logs, they can be resummed. Magnea and Sterman and '90
- We can avoid the π^2 terms by choosing a time-like value $\mu^2 = -q^2$

$$C_S(-q^2, -q^2) = 1 + 0.393\,\alpha_s(-q^2) - 0.152\,\alpha_s^2(-q^2) + \ldots$$

  - same expansion coefficients as $C_S(Q^2, Q^2)$
- Note: RG-evolution defines $\alpha_s(\mu^2)$ for *any* μ^2

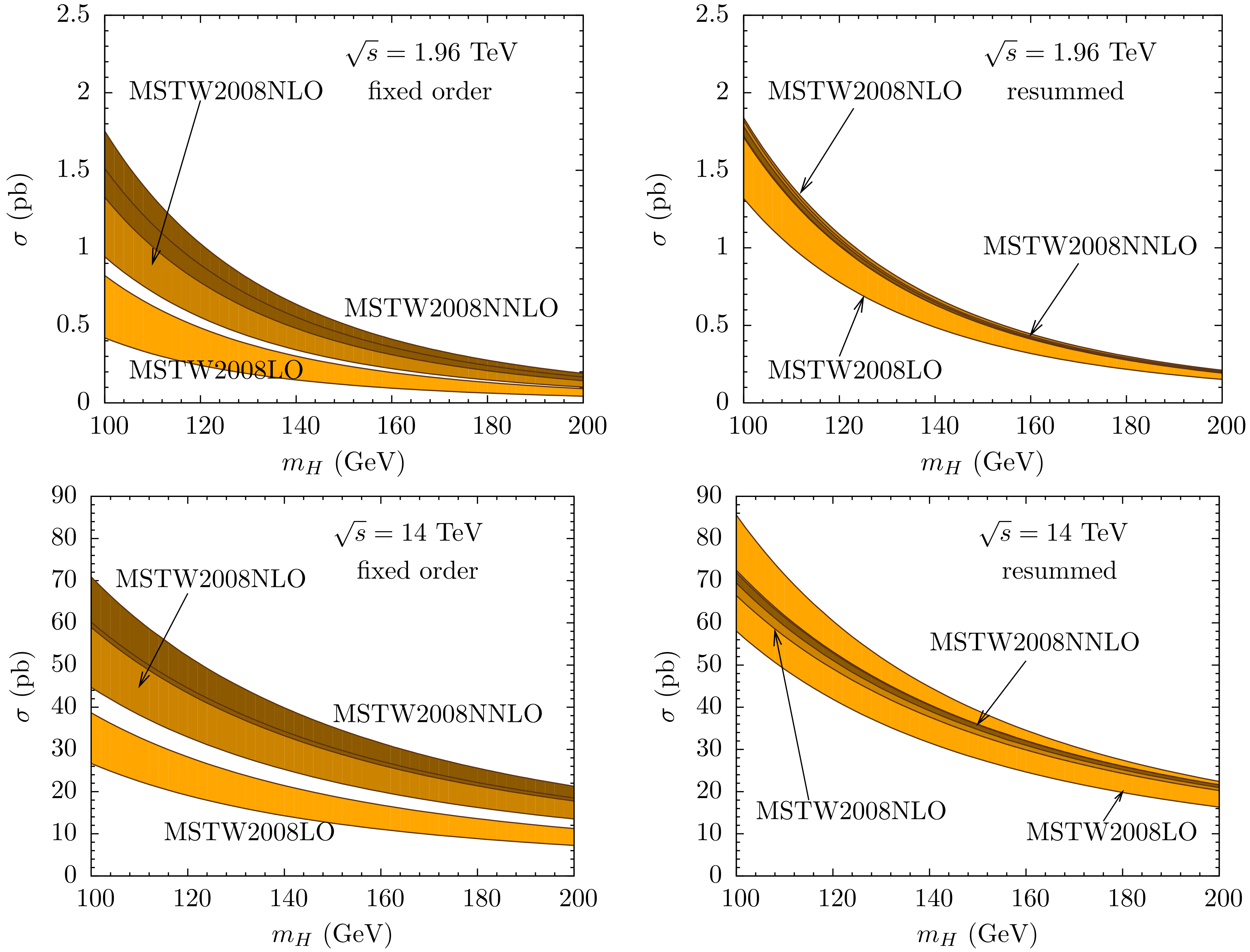
Time-like vs. space-like μ^2

Heinrich et al. ’09; Baikov et al.; T. Gehrmann et al. ’10



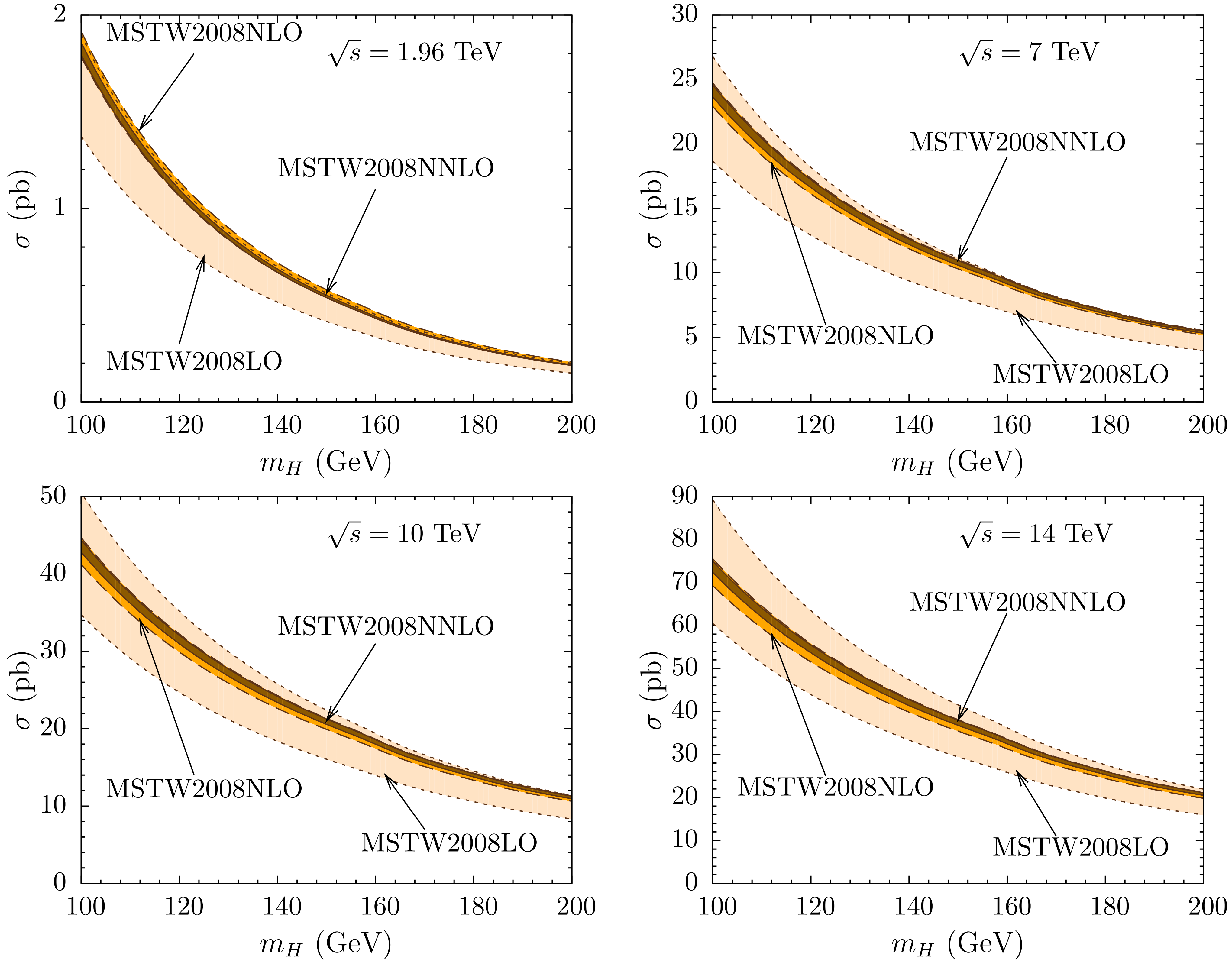
- Convergence is much better for $\mu^2 < 0$
- Evaluate *H* for $\mu^2 < 0$ where convergence is good and use RG to evolve to arbitrary scale

Results, scale variation



no large *K*-factor!

Update with EW corrections



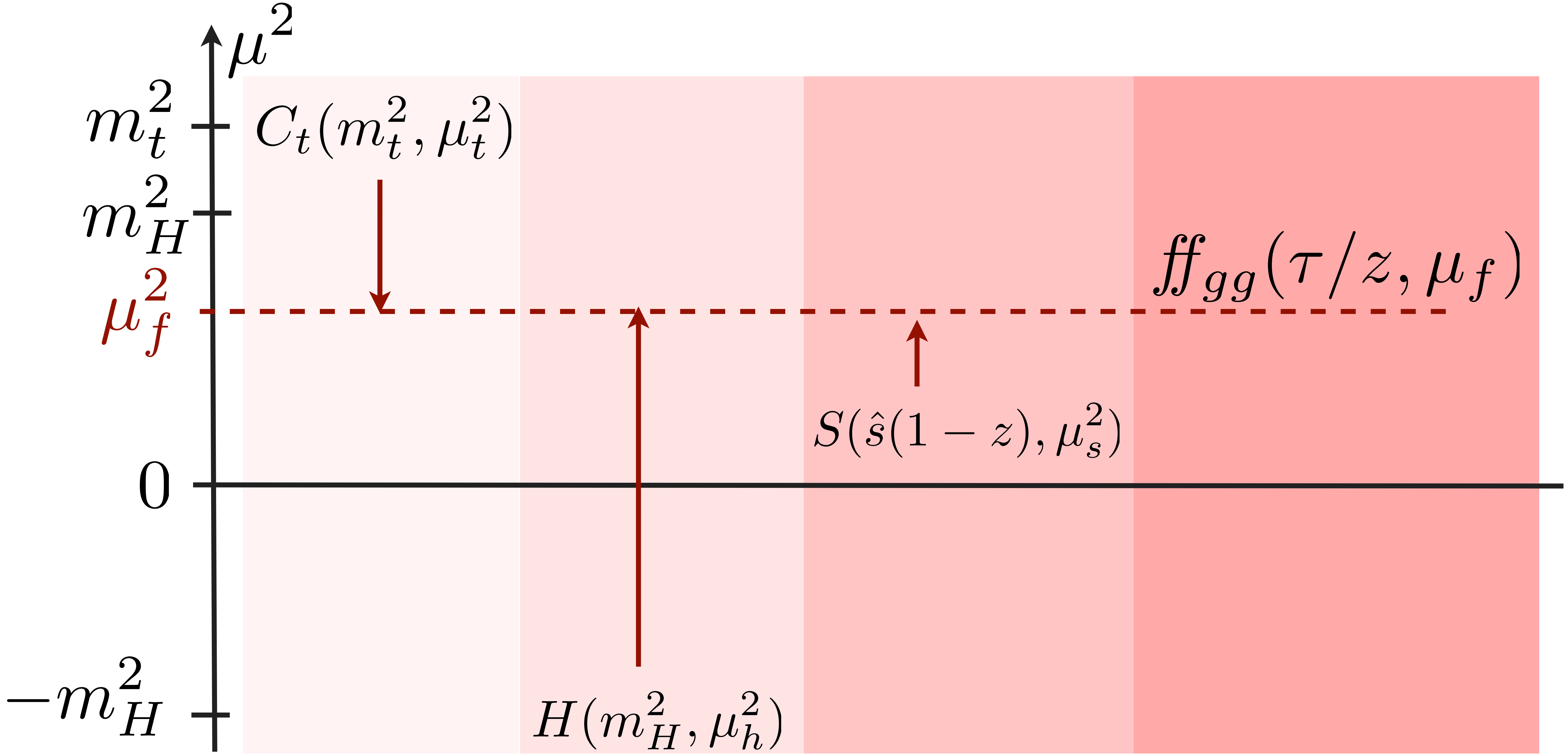
Conclusions

- Higgs production cross section is not strongly dominated by partonic threshold.
  - Significant scheme dependence in soft-gluon resummation.
- Large corrections in hard function
  - Much better convergence if it is evaluated for space-like kinematics, setting $\mu^2 =- M_H^2$
- Both soft-gluon and π^2 resummation increase cross section.

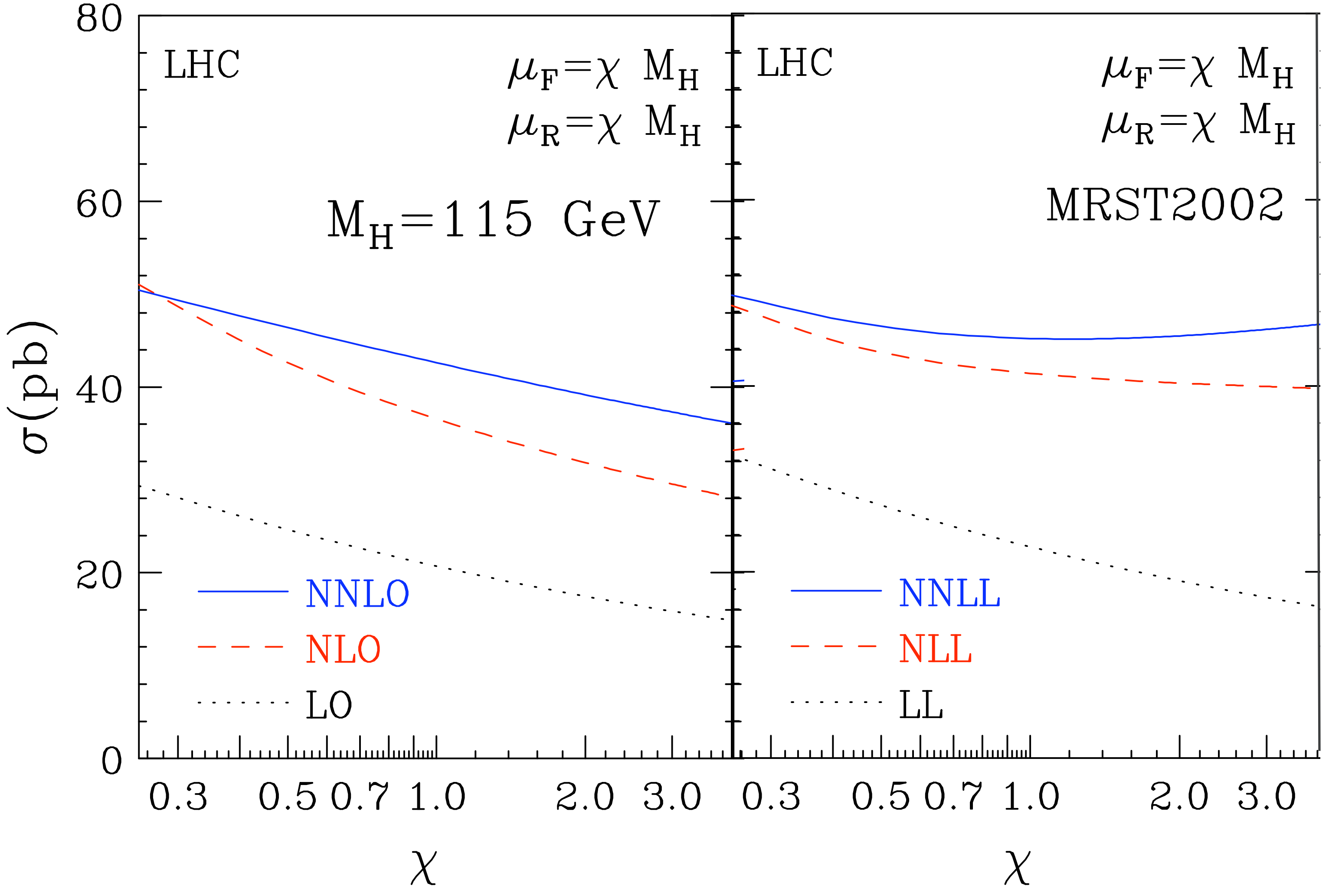
Extra

Resummation by RG evolution

- Evaluate each part at its characteristic scale, evolve to common scale:



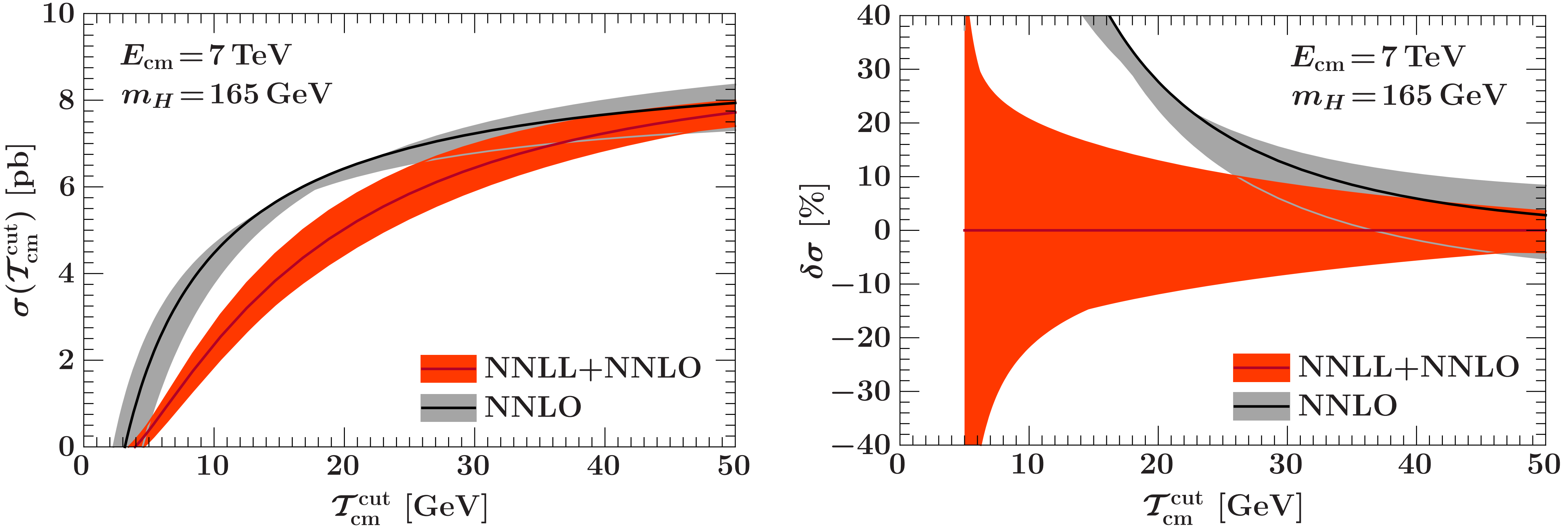
Traditional soft-gluon resummation



Catani, de Florian, Grazzini, Nason ’03
numerical update: de Florian and Grazzini ’09

Beam thrust as jet veto

Berger, Marcantonini, Stewart, Tackmann and Waalewijn '11



- Small uncertainties of fixed-order calculation are misleading, arise from cancellation of large hard and collinear corrections.